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A BAYESIAN APROACH FOR ZERO-INFLATED COUNT DATA IN PSYCHOLOGICAL RESEARCH

Naci Murat*[1], Mehmet Ali[2] Cengiz, Bozkurt Koç[3]

[1]Ondokuz Mayıs University, Department of Industrial Engineering, Turkey
[2]Ondokuz Mayıs University, Department of Statistics, Turkey
[3]Ondokuz Mayıs University, Department of Psychology, Turkey
*Corresponding author: nacimurat@omu.edu.tr

ABSTRACT

Count data in psychological research are commonly modelled using zero-inflated Poisson regression. This model can be viewed as a latent mixture of an "always-zero" component and a Poisson component. In this study we introduce a Bayesian approach for zero inflated Poisson model, and discuss model comparisons and the interpretation of their parameters. As illustrated with two real-world examples psychological research, both Bayesian and classic approach of models can easily be fitted with a great gain.

Keywords: Bayesian modelling, Zero inflated models, Psychological research

1. INTRODUCTION

Count data such as the number of steps taken towards separation and divorce and the number of post-breakup unwanted pursuit behaviours can occur in many psychological researches [1, 2]. Such count data are typically much skewed and exhibit a lot of zero count observations. Although poison regression models are standard method for count data, it is well known that analysing these data using standart poisson modelsear models is inappropriate [3]. Standart Poisson regression models have several diffuculties. First, it assumes that the mean and the variance are equal. However, count data often exhibit larger variance than predicted by the mean (overdispersion). Recently, to overcome this problem, Zero Inflated Poisson (ZIP) model were used for such data.

Although recent studies in many areas have discussed the advantages of the use of the zero inflated Poisson, that were first introduced by [4] only a few studies in psychological research discuss the use of such models [5] consider zero-inflated Poisson (ZIP) models and discuss model comparisons and the interpretation of their parameters with the outcome of interest counting the number of unwanted pursuit behaviours in ex-partners, are used to illustrate model fit and interpretation of parameters.

A Bayesian approach is a useful tool in statistics where all forms of uncertainty are expressed in terms of probability. The advantages of Bayesian inference are well known and include elicitation of prior beliefs, avoidance of asymptotic approximations and practical estimation of functions of parameters [6, 7].

Several authors have recently proposed Bayesian alternatives to fitting zero inflated models. [8] developed a Bayesian approach for generalized Poisson regression models by Markov Chain Monte Carlo (MCMC) method; [9] discussed Bayesian inference of ZIP model by using MC integration with important sampling to analyze fetal movement data, [10] studied zero-inflated distributions with the Bayesian point of view using the data augmentation algorithm, [11] developed a full Bayesian estimation method of zero-inflated regression model by Gibbs sampling, [12] developed some Bayesian count data models which were combined with semiparametrically structured additive predictors, [13] extends the zero-inflated mixed regression model to a very general semiparametric mixed-effects models for zero-inflated count data with a Bayesian approach and [14] analyze zero-inflated models with a Bayesian approach to estimate regression parameters and compare prediction performances with the frequentist approach for applications to road safety countermeasures.

In Bayesian approach, interest lies in estimating posterior distributions of model parameters rather than individual parameter values and asymptotic standard errors. Nevertheless, iterative computational algorithms may still be used to produce a sequence of parameter values. However, in the Bayesian setting, convergence assessment involves checking that the sequence, or chain, has converged to and provides a representative sample from the posterior distribution. Despite the growing popularity of MCMC methods in Bayesian approach, the use of MCMC convergence diagnostics is still relatively uncommon. Tools for assessing convergence are already available for many statistical models [15]. For conditions that govern Markov chain convergence and rates of convergence [16-25]. Convergence diagnostics are rarely used in zero inflated models for medical applications.

Despite the growing popularity of Bayesian approach, specifically MCMC methods for zero inflated poisson model in many medical areas, the use of MCMC methods of ZIP models in psychological research is still relatively unknown. In this study we introduce A Bayesian approach for the zero-inflated Poisson model with two real-world data sets in psychological research. First is the outcome of interest counting the number of unwanted pursuit behaviours in ex-partners (the data used by [5]) and the second is the outcome of the number of types of violent behaviours in teenagers.

For the first data set, [5] consider zero-inflated Poisson (ZIP) models and discuss model comparisons and the interpretation of their parameters for the Unwanted Pursuit Behaviours (UPB) data set. We reconsider ZIP model with a Bayesian approach and compares the results of ZIP model and ZIP model with a Bayesian approach for the same data used by [5]. For the second data set we introduce a Bayesian approach for ZIP model with the outcome of the number of the types of violent behaviours in teenagers. We compare standart and Bayesian ZIP models for this application.

2. MATERIALS AND METHODS

2.1. First motivating example 1: Modelling unwanted pursuit behaviour

Our first motivating example is a subsample of the Interdisciplinary Project for the Optimization of Separation (IPOS) trajectories conducted in Flanders (http://www.scheidingsonderzoek.be) that aims to gain insight into separation trajectories. A part of the data was used [5]. More specifically, They focus on a sample of 387 participants who responded to an adapted version of a Relational Pursuit-Pursuer Short Form (RP-PSF) [26] used to assess the extent of unwanted pursuit behaviour (UPB) perpetrations displayed since the time the couple broke up. The total of 28 items (ranging from 'leaving unwanted gifts' to 'threatening to hurt yourself'), each measured on a five-point Likert scale (from 0 = never to 4 = over five times), was used as an overall index of perpetration (with higher scores indicating higher levels of perpetrations). A participant who answered 'never' to all these 28 UPB items will have an UPB count equal to 0, while a participant who answered 'over five times' to 'leaving unwanted gifts' and 'never' to all other items will, for example, have an UPB count equal to 4. [5] explore the impact of two predictors on this outcome: a binary indicator for 'education level' (0 = lower than bachelor's degree, or 1 = at least bachelor's degree), and a continuous measurement for the level of 'anxious attachment' in the former partner relationship. The latter was measured using a total of five anxious attachment items with results normalized to a z-score. A more in-depth psychological review of the UPB data is described in [2].

2.2. Second motivating example : Violence at school

Our second motivating example is a subsample of survey of the violence in school conducted by [27]. In general violence is defined as "the intentional use of physical force or power, threatened or actual, against oneself, another person, or against a group or community, that either result in or has a high likelihood of resulting in injury, death, psychological harm, maldevelopment or deprivation" [28]. School violence is an issue that is posing severe harm to most of the schools in the word. Schools in all around the world are gravely affected by this problem. The peculiar thing about school violence is that it has taken many forms over the years. If the authorities try to arrest one type of school violence then another type arises from the corner. It is indeed very difficult for a normal school administration to take control of the situation if they do not understand the various types of school violence. It is important to know what school violence is for understanding the school violence classifications The various types of the school violence can be then be analyzed to understand how one can protect their children against all odd in educating them. The school violence can occur in several ways as follow [29, 30, 31].

- To harass or tease other students
- To indulge in vandalism
- Sexual harassment
- Bringing knives and other harmful weapons
- Verbal abuse
- Physical attacks
- Fights resulting in injury
- Fighting with a member of a gang

School violence is a many-faceted problem, making it difficult for researchers and practitioners to pinpoint its causes [32]. In general biological, personal, psychological, familial, environmental, social, and cultural factors are effective in emergence of school violence [33, 34].

The study sample consists of a total of 1381 students from 15 high schools with different socio-economic and cultural levels. [27] Investigated which factors have effect on the violence and type of violence in school.

2.3. A bayesian approach for zero inflated poisson model

Let $Y_i$zeros are assumed to arise in two ways corresponding to distinct underlying states. The first state occurs with probability ω and produces only zeros, while the other state occurs with probability $1-\omega$ and leads to a standard Poisson distribution with mean λ_i [35]. Then $Y_i$follows a ZIP distribution with the following pmf:

$$(y_i;\lambda_i,\omega)=\begin{cases} \omega+(1-\omega)e^{-\lambda_i}, & y_i=0\,, \\ (1-\omega)\frac{e^{-\lambda_i}\lambda_i^{y_i}}{y_i!} \quad , & y_i=1,2,\ldots\,, \end{cases} \quad (1)$$

where $0 \le \omega \le 1$.Using the log link function by letting $log\lambda = x_i'\beta$ where β is a vector of unknown regression coefficients and x_i' is the design matrix with covariates, the likelihood functions of two zero-inflated regression models are given respectively by

$$(\beta, L\omega) = \prod_{i=1}^{n}\left[I_{(y_i=0)}\{\omega + (1-\omega)e^{-\lambda_i}\} + I_{(y_i>0)}(1-\omega)\frac{e^{-\lambda_i}\lambda_i^{y_i}}{y_i!}\right], \quad (2)$$

Bayesian statistical conclusions about a parameter θ are made in terms of probability statements. These probability statements are conditional on the observed value of y, and in our notation are simply written as $\pi(\theta/y)$. The posterior distribution can be written as $\pi(\theta/y) \propto L(\theta;y)\pi(\theta)$where $L(\theta;Y)$ is the likelihood function and $\pi(\theta)$ is the prior distribution. Under Bayesian framework, we need to specify prior distributions for unknown parameters in the models as in Table 2.

The Markov chain Monte Carlo (MCMC) method is a general simulation method for sampling from posterior distributions and computing posterior quantities of interest. MCMC methods sample successively from a target distribution. Each sample depends on the previous one, hence the notion of the Markov chain. The Markov chain method has been quite successful in modern Bayesian computing. Only in the simplest Bayesian models can you recognize the analytical forms of the posterior distributions and summarize inferences directly. In moderately complex models, posterior densities are too difficult to work with directly. With the MCMC method, it is possible to generate samples from an arbitrary posterior density and to use these samples to approximate expectations of quantities of interest. Several other aspects of the Markov chain method also contributed to its success. Most importantly, if the simulation algorithm is implemented correctly, the Markov chain is guaranteed to converge to the target distribution under rather broad conditions, regardless of where the chain was initialized. Properties of Markov chains are discussed in [6, 19, 20, 24, 36-43].

This study includes several statistical diagnostic tests that can help you assess Markov chain convergence [44], tests whether the mean estimates have converged by comparing means from the early and latter part of the Markov chain. Two-sided test based on a z score statistic. Large absolute z values indicate rejection. Heidelberger-Welch (stationarity test) tests whether the Markov chain is a covariance (or weakly) stationary process. Failure could indicate that a longer Markov chain is needed One-sided test based on a Cramer–von Mises statistic. Small p-values indicate rejection. Heidelberger-Welch (half-width test) Reports whether the sample size is adequate to meet the required accuracy for the mean estimate. Failure could indicate that a longer Markov chain is needed. If a relative half-width statistic is greater than a predetermined accuracy measure, this indicates rejection [45-48] evaluates the accuracy of the estimated (desired) percentiles by reporting the number of samples needed to reach the desired accuracy of the percentiles. Failure could indicate that a longer Markov chain is needed. If the total samples needed are fever than the Markov chain sample, this indicates rejection. Autocorrelation measures dependency among Markov chain samples. High correlations between long lags indicate poor mixing. Effective sample size [49] relates to autocorrelation; measures mixing of the Markov chain. Large discrepancy between the effective sample size and the simulation sample size indicates poor mixing.

3. RESULTS

3.1. The unwanted pursuit behaviour dataset results

In this example, we use zero-inflated Poisson model with a Bayesian approach to investigate the impact of 'education level' and 'level of anxious attachment' on the number of unwanted pursuit behaviour (UPB) perpetrations in the context of couple separation trajectorie

For this study, a random walk Metropolis algorithm for all parameters is used. The priors on all parameters are assumed to be Normal (0,var=1000). Before using the Bayesian approach results, it is needed checking the convergence assessment, that involves checking that the sequence, or chain, has converged to and provides a representative sample from the posterior distribution. Table 1 shows the results for the convergence criterias such as Thumb rule, Geweke, Heidelberger-Welch (stationarity test), Heidelberger-Welch (half-width test), Raftery-Lewis and Effective sample size.

According to the diagnostic statistics in Table 1 the Markov chain has reached convergence for each parameter for all models using six different convergence methods. Geweke diagnostics are not significant for each parameter (all p-values>0.05). Heidelberger-Welch stationary test shows that none of the p-values indicate rejection for convergence for each parameter for each model. For the Heidelberger-Welch half-width test each parameter showed no indication of rejection for convergence for all models. The results of Raftery-Lewis diagnostics indicate no problem with each parameter since the total samples needed are less than the Markov chain sample. Monte Carlo standard errors show that the standard error of the mean estimates for each parameter is relatively small with respect to the posterior standard deviations.

We produce a number of graphs which also aid convergence diagnostic checks. As an example Fig. 1 shows diagnostic plots for education (beta1). From the trace plots we can say that the mean of the Markov chain has stabilized and appears constant over the graphs. The plots show that the chains appear to have reached convergence. The posterior autocorrelations are quite small and the posterior density appears bell-shaped.

Table 1. Results for convergence diagnostics for each parameter in all models used

	Thumb Rule	Geweke Diagnostics	Raftery-Lewis Diagnostics	Heidelberger-Welch Diagnostics	Effective Sample Sizes	
Parameter	MCSE/SD	Pr > |z|	Dependence Factor	Stationarity Test	Half-width Test	Efficiency
Count Component						
Intercept	0.0118	0.3886	1.1642	Passed	Passed	0.7213
Education	0.0116	0.392	1.1207	Passed	Passed	0.7387
Anxious	0.0118	0.4566	1.1207	Passed	Passed	0.7176
Zero Component						
Intercept	0.0114	0.373	1.1676	Passed	Passed	0.7685
Education	0.0114	0.1696	1.1773	Passed	Passed	0.7737
Anxious	0.0116	0.0543	1.1484	Passed	Passed	0.7432

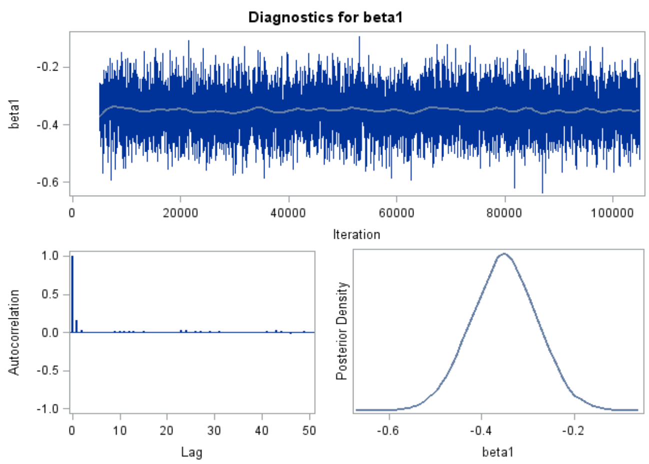

Fig. 1: Diagnostic plots for education

Therefore, we assume that after a specific number of iterations, the chain has reached its target distribution and we can use the good samples for posterior inference. Summary statistics for each model for Zero-inflated regression with a Bayesian approach are presented in Table 2.

Table 2. Summary statistics for each parameter in all models used

	Means	S.D.	MCSE	HPD credible intervals	
Count Component					
Intercept	1.9189	0.0444	0.0005	1.8891	1.9493
Education	-0.3512	0.0712	0.0008	-0.3989	-0.3035
Anxious	0.1332	0.0345	0.0004	0.1096	0.1564
Zero Component					
Intercept	0.6765	0.1421	0.0016	0.579	0.7725
Education	-0.2322	0.2194	0.0024	-0.3793	-0.0813
Anxious	-0.4885	0.1119	0.0013	-0.5631	-0.4154

For Bayesian zero-inflated model Table 2 reports posterior means, standard deviations and Highest Posterior Density (HPD) credible intervals for each parameter. From Table 2 it is easy to say that Bayesian ZIP gives the similar results with classical ZIP model in terms of parameter estimations. Since HPD credible interval values for all parameters do not consist of zero, the effect of both education level and anxious level on the number of unwanted pursuit behaviour (UPB) perpetrations is highly significant. Table 3 consists of the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) values for model comparison.

Table 3. Model comparisons

Approach	AIC	BIC
ZIP	1616.9	1614.5
Bayesian ZIP	1205.3	1198.7

The AIC, BIC and LogL values for Bayesian ZIP model suggests a much improved fit over the classic ZIP model (gives smaller the AIC and BIC values).

3.2. Violence at school example results

In this example, we use zero-inflated Poisson model with a Bayesian approach to investigate the impact of 'gender', 'level of mother education', 'level of father education', 'violence' and 'violence at home' on the number of types of violence at school. First we use standart zero inflated Poisson model. Results of standart zero inflated Poisson model are given Table 4.

The analysis of parameter estimates results show that the effect of Gender, Father's Education, Violence and Violence at home on the number of types of violence at school is highly significant and the effect of Mother Education on the number of types of violence at school is insignificant at the 5% level. Before using the Bayesian approach results, it is needed checking the convergence assessment.

Table 4. Results of classical zero inflated Poisson model

Parameter	Means	S.E.	p>|z|
Count Component			
Intercept	1.2329	0.3284	0
Gender	0.5921	0.114	0
Mother education	0.0015	0.035	0.966
Father education	-0.0795	0.038	0.037
Violence	-0.8646	0.1201	0
Violence at home	-0.5111	0.0981	0
Zero Component	-0.1661	0.1233	0.178

Table 5 shows the results for the convergence criterias such as Thumb rule, Geweke, Heidelberger-Welch (stationarity test), Heidelberger-Welch (half-width test), Raftery-Lewis and Effective sample size.

The diagnostic statistics in Table 5 shows that the Markov chain has reached convergence for each parameter for all models using six different convergence methods. As an example, Fig. 2 shows diagnostic plots for mother education (beta2). From the trace plots, we can say that the mean of the Markov chain has stabilized and appears constant over the graphs. The plots show that the chains appear to have reached convergence. The posterior autocorrelations are quite small and the posterior density appears bell-shaped.

Table 5. Results for convergence diagnostics for each parameter in all models used

	Thumb Rule	Geweke Diagnostics	Raftery-Lewis Diagnostics	Heidelberger-Welch Diagnostics	Effective Sample Sizes	
Parameter	MCSE/SD	Pr > |z|	Dependence Factor	Stationarity Test	Half-width Test	Efficiency
Count Component						
Intercept	0.0162	0.7247	1.4391	Passed	Passed	0.3807
Gender	0.0152	0.3092	1.3564	Passed	Passed	0.4302
Mother education	0.0152	0.8671	1.3564	Passed	Passed	0.4332
Father education	0.0148	0.0835	1.3449	Passed	Passed	0.4581
Violence	0.0159	0.253	1.3793	Passed	Passed	0.3959
Violence at home	0.0154	0.9594	1.4146	Passed	Passed	0.4218
Zero Component	0.0111	0.4326	1.0755	Passed	Passed	0.813

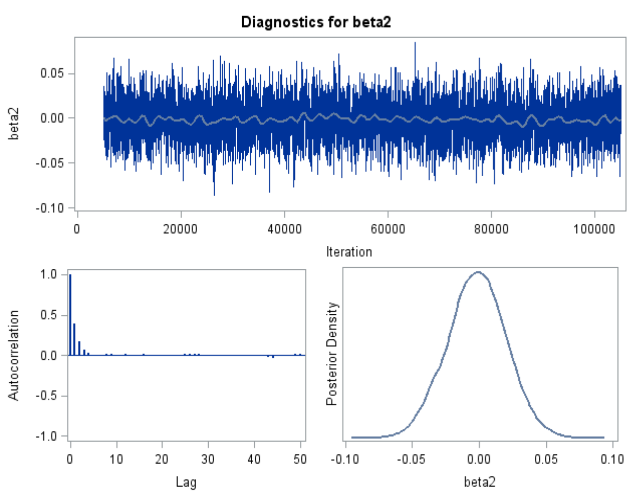

Fig.2. Diagnostic plots for mother education

Therefore we assume that after a specific number of iterations. The chain has reached its target distribution and we can use the good samples for posterior inference. Summary statistics for each model for Zero-inflated regression with a Bayesian approach are presented in Table 6.

Table 6.Results of Parameters Estimations

Parameter	Means	S.D.	MCSE	HPD credible intervals	
Count Component					
Intercept	1.2271	0.103	0.0017	1.1578	1.2961
Gender	-0.137	0.029	0.0004	-0.157	-0.118
Mother education	-0.002	0.022	0.0003	-0.016	-0.013
Father education	-0.247	0.022	0.0003	-0.262	-0.232
Violence	-0.101	0.023	0.0004	-0.117	-0.085
Violence at home	0.0261	0.018	0.0003	0.0138	0.0385
Zero Component	0.6249	0.016	0.0035	0.9943	1.6025

For Bayesian zero-inflated model, Table 6 reports posterior means, standard deviations and Highest Posterior Density (HPD) credible intervals for each parameter. From Table 6 it is easy to say that Bayesian ZIP gives the similar results, except for mother education, with classical ZIP model in terms of parameter estimations. Since HPD credible interval values for all parameters do not consist of zero, the effect of Gender, Mother education, Father education, Violence and Violence at home on the number of types of violence at school are highly significant. However, the effect of Mother education on the number of types of violence at school is insignificant at the 5% level for classic zero inflated Poisson model.

Table 7 consists of the Akaike Information Criteria (AIC), the Bayesian Information Criteria (BIC) values and LogL for model comparison.

Table 7. Model comparisons

Approach	AIC	BIC
ZIP	1341.87	1317.98
Bayesian ZIP	965.73	951.24

The AIC, BIC and LogL values for Bayesian ZIP model suggests a much improved fit over the classic ZIP model. (gives smaller the AIC and BIC values)

Table 8. Comparison of diagnostics

Diagnostics	Number of iterations
Thumb Rule	12100
Geweke	13500
Raftery-Lewis Diagnostics	13670
Heidelberger-Welch (stationary)	14100
Heidelberger-Welch (half-widht)	14100
Effective Sample Sizes	12800

4. CONCLUSION

In this study, we presented some details of Bayesian ZIP models with two real life psychological data sets. For both applications, several statistical diagnostic tests, that can help you assess Markov chain convergence, were presented. After showing that the Markov chain has reached convergence for each parameter for all models using six different convergence methods, we analysis the data sets using ZIP model with a Bayesian approach and compares standart and Bayesian ZIP models in terms parameter estimations and information criterias such as AIC and BIC. Bayesian ZIP model suggests a much improved fit over the classic ZIP model both applications.

In the Bayesian approach, convergence assessment involves checking that the sequence, or chain, has converged to and provides a representative sample from the posterior distribution. Despite the growing popularity of MCMC methods in Bayesian approach, the use of MCMC convergence diagnostics is still relatively uncommon. Therefore we used six different convergence methods for both applications and compared those convergence methods for violence at school data set. While Bayesian ZIP models are seldom used in psychological research, we hope in the future that Bayesian ZIP models will be use as an alternative for classic ZIP model in psychological research.

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